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Radiatively Generated Isospin Violations in the Nucleon and the NuTeV Anomaly

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Abstract

Predictions of isospin asymmetries of valence and sea distributions are presented which are generated by QED leading $\mathcal{O}(\alpha)$ photon bremsstrahlung effects. Together with isospin violations arising from nonperturbative hadronic sources (such as quark and target mass differences) as well as with even a conservative contribution from a strangeness asymmetry ($s \neq \bar{s}$), the discrepancy between the large NuTeV 'anomaly' result for $\sin^2\theta_W$ and the world average of other measurements is removed.

The NuTeV collaboration recently reported [1] a measurement of the Weinberg angle $s_W^2 \equiv \sin^2\theta_W$ which is approximately three standard deviations above the world average [2] of other electroweak measurements. Possible sources for this discrepancy (see, for example, [3, 4, 5, 6, 7]) include, among other things, isospin-symmetry violating contributions of the parton distributions in the nucleon, i.e., nonvanishing δq_v and $\delta\bar{q}$ defined via

$$
\begin{aligned}
\delta u_v(x,Q^2) &= u_v^p(x,Q^2) - d_v^n(x,Q^2) \\
\delta d_v(x,Q^2) &= d_v^p(x,Q^2) - u_v^n(x,Q^2)
\end{aligned}
\tag{1}
$$

where $q_v = q - \bar{q}$ and with analogous definitions for $\delta\bar{u}$ and $\delta\bar{d}$. The valence asymmetries δu_v and δd_v were estimated within the nonperturbative framework of the bag model [4, 5, 8, 9, 10] and resulted in a reduction of the above mentioned discrepancy by about 30%. It should be emphasized that these nonperturbative charge symmetry violating contributions arise predominantly through mass differences $\delta m = m_d - m_u$ of the struck quark and from target mass corrections related to $\delta M = M_n - M_p$.

The additional contribution to the valence isospin asymmetries stemming from radiative QED effects was presented recently [11]. Following the spirit of this publication we shall evaluate δq_v and $\delta\bar{q}$ in a slightly modified way based on the approach presented in [12, 13] utilizing the QED $\mathcal{O}(\alpha)$ evolution equations for $\delta q_v(x,Q^2)$ and $\delta\bar{q}(x,Q^2)$ induced by the photon radiation off the (anti)quarks. To *leading* order in α we have

$$
\begin{aligned}
\frac{d}{d\ln Q^2}\,\delta u_v(x,Q^2) &= \frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\,P\left(\frac{x}{y}\right) u_v(y,Q^2) \\
\frac{d}{d\ln Q^2}\,\delta d_v(x,Q^2) &= -\frac{\alpha}{2\pi}\int_x^1 \frac{dy}{y}\,P\left(\frac{x}{y}\right) d_v(y,Q^2)
\end{aligned}
\tag{2}
$$

with $P(z) = (e_u^2 - e_d^2)P_{qq}^\gamma(z) = (e_u^2 - e_d^2)\left(\frac{1+z^2}{1-z}\right)_+$, and similar evolution equations hold for the isospin asymmetries of sea quarks $\delta\bar{u}(x,Q^2)$ and $\delta\bar{d}(x,Q^2)$. Notice that the addition [11, 14] of further terms proportional to $(\alpha/2\pi)e_q^2 P_{q\gamma} * \gamma$ to the r.h.s. of (2) would actually amount to a subleading $\mathcal{O}(\alpha^2)$ contribution since the photon distribution $\gamma(x,Q^2)$ of the

nucleon is of $\mathcal{O}(\alpha)$ [15, 16, 17, 18, 19, 20]. We integrate (2) as follows:

$$\begin{aligned}\delta u_v(x,Q^2) &= \frac{\alpha}{2\pi}\int_{m_q^2}^{Q^2} d\ln q^2 \int_x^1 \frac{dy}{y}\; P\left(\frac{x}{y}\right) u_v(y,\,q^2)\\ \delta d_v(x,Q^2) &= -\frac{\alpha}{2\pi}\int_{m_q^2}^{Q^2} d\ln q^2 \int_x^1 \frac{dy}{y}\; P\left(\frac{x}{y}\right) d_v(y,\,q^2)\end{aligned}\tag{3}$$

and similarly for $\delta\bar{u}$ and $\delta\bar{d}$ utilizing the usual isospin symmetric leading–order (LO) parton distributions $q_v(x,q^2)$ and $\bar{q}(x,q^2)$ of the dynamical (radiative) parton model [21]. The current quark mass m_q being the usual kinematical lower bound for a photon emitted by a quark – similar to the electron mass m_e for a photon radiated off an electron [22]. Here we conservatively choose $m_q = 10$ MeV, i.e., of the order of the current quark masses [2]. The parton distributions at $q^2 < \mu_{\rm LO}^2$ in (3), where $\mu_{\rm LO}^2 = 0.26$ GeV^2 is the input scale in [21], are taken to equal their values at the perturbative input scale $\mu_{\rm LO}^2$, $\overset{(-)}{q}(y,\,q^2 \leq \mu_{\rm LO}^2) = \overset{(-)}{q}(y,\,\mu_{\rm LO}^2)$, i.e. are 'frozen'.

The resulting valence isospin asymmetries δu_v and δd_v at $Q^2 = 10$ GeV^2 are presented in Fig. 1 where they are compared with the corresponding nonperturbative bag model results [5], with the latter ones being of entirely different origin, i.e., arising dominantly through the mass differences δm and δM. As can be seen, our radiative QED predictions and the bag model estimates are comparable for δu_v but differ considerably for δd_v. It should furthermore be noted that, although our method differs somewhat from that in [11], our resulting $\delta q_v(x,Q^2)$ turn out to be quite similar, as already anticipated in [11].

Going beyond the results in [4, 5, 8, 9, 10] and [11] we present in Fig. 2 our estimates for the isospin violating sea distributions for $\delta\bar{u}$ and $\delta\bar{d}$ at $Q^2 = 10$ GeV^2. Similar results are obtained for the LO CTEQ4 parton distributions [23] where also valence–like sea distributions are employed at the input scale $Q_0^2 = 0.49$ GeV^2, i.e., $x\bar{q}(x,Q_0^2) \to 0$ as $x \to 0$. Such predictions may be tested by dedicated precision measurements of Drell–Yan and DIS processes employing neutron (deuteron) targets as well.

Turning now to the impact of our $\delta\overset{(-)}{q}(x,Q^2)$ on the NuTeV anomaly, we present in

Table I the implied corrections Δs_W^2 to s_W^2 evaluated according to

$$\Delta s_W^2 = \int_0^1 F[s_W^2,\, \delta \overset{(-)}{q};\, x]\, \delta \overset{(-)}{q}(x, Q^2)\, dx \tag{4}$$

at $Q^2 \simeq 10$ GeV^2, appropriate for the NuTeV experiment. The functionals $F[s_W^2,\, \delta \overset{(-)}{q};\, x]$ are presented in [3] according to the experimental methods [1] used for the extraction of s_W^2 from measurements of

$$R^{\nu(\bar{\nu})}(x, Q^2) \equiv d^2\sigma_{NC}^{\nu(\bar{\nu})N}(x, Q^2)/d^2\sigma_{CC}^{\nu(\bar{\nu})N}(x, Q^2)\,. \tag{5}$$

Since the isospin violation generated by the QED $\mathcal{O}(\alpha)$ correction is such as to remove more momentum from up–quarks than down–quarks, as is evident from Fig. 1, it works in the right direction to reduce the NuTeV anomaly [1], i.e., $\sin^2\theta_W = 0.2277 \pm 0.0013 \pm 0.0009$ as compared to the world average of other measurements [2] $\sin^2\theta_W = 0.2228(4)$. Also shown in Table I are the *additional* contributions to Δs_W^2 stemming from the nonperturbative hadronic bag model calculations [4, 5, 8, 9, 10] where isospin symmetry violations arise predominantly through the quark and target mass differences δm and δM, respectively, as mentioned earlier. These contributions are comparable in size to our radiative QED results.

Although the NuTeV group [1] has taken into account several uncertainties in their original analysis due to a nonisoscalar target, higher–twists, charm production, etc., they have disregarded, besides isospin violations, effects caused by the strange sea asymmetry $s \neq \bar{s}$. Recent nonperturbative estimates [7, 24, 25, 26] resulted in sizeable contributions to Δs_W^2 similar to the ones in Table I. As a conservative estimate we use [25] $\Delta s_W^2|_{\rm strange} = -0.0017$. With the results in Table I, the *total* correction therefore becomes

$$\begin{aligned} \Delta s_W^2|_{\rm total} &= \Delta s_W^2|_{\rm QED} + \Delta s_W^2|_{\rm bag} + \Delta s_W^2|_{\rm strange} \\ &= -0.0011 \;-\; 0.0015 \;-\; 0.0017 \\ &= -0.0043\,. \end{aligned} \tag{6}$$

Δs_W^2	δu_v	δd_v	$\delta \bar{u}$	$\delta \bar{d}$	total
QED	-0.00071	-0.00033	-0.000019	-0.000023	-0.0011
bag	-0.00065	-0.00081	—	—	-0.0015

Table 1: The QED corrections to Δs_W^2 evaluated according to (4) using (3). The nonperturbative bag model estimates [9] are taken from [5]; different nonperturbative approaches give similar results [5].

Thus the NuTeV measurement ('anomaly') of $\sin^2\theta_W = 0.2277(16)$ will be shifted to $\sin^2\theta_W = 0.02234(16)$ which is in agreement with the standard value 0.2228(4).

Finally, it should be mentioned that, for reasons of simplicity, it has become common (e.g. [6, 7, 11, 24, 26]) to use the Paschos–Wolfenstein relation [27] for an isoscalar target, $R^-_{\mathrm{PW}} = \frac{1}{2} - s_W^2$, for estimating the corrections discussed above,

$$R^- \equiv \frac{\sigma_{\mathrm{NC}}^{\nu N} - \sigma_{\mathrm{NC}}^{\bar\nu N}}{\sigma_{\mathrm{CC}}^{\nu N} - \sigma_{\mathrm{CC}}^{\bar\nu N}} = R^-_{\mathrm{PW}} + \delta R^-_I + \delta R^-_s\,, \tag{7}$$

instead of the experimentally directly measured and analyzed ratios $R^{\nu(\bar\nu)}$ in (5), where [3]

$$\delta R^-_I \simeq \left(\frac{1}{2} - \frac{7}{6}s_W^2\right)\frac{\delta U_v - \delta D_v}{U_v + D_v}\,, \quad \delta R^-_s \simeq -\left(1 - \frac{7}{3}s_W^2\right)\frac{S^-}{U_v + D_v} \tag{8}$$

with $Q_v(Q^2) = \int_0^1 x\, q_v(x,Q^2)\,dx$, $\delta\, Q_v(Q^2) = \int_0^1 x\,\delta\, q_v(x,Q^2)\,dx$ and $S^-(Q^2) = \int_0^1 x[s(x,Q^2) - \bar{s}(x,Q^2)]\,dx$. (Note that the correct expressions for *both* δR^-_I and δR^-_s have been presented only in [3]). Our radiative QED results in Fig. 1 imply $\delta U_v = -0.002226$ and $\delta D_v = 0.000890$ which, together with $U_v + D_v = 0.3648$, give $\Delta s_W^2|_{\mathrm{QED}} = \delta R^-_I|_{\mathrm{QED}} = -0.002$ according to (8), whereas the correct value in Table I is only *half* as large. Similar overestimates are obtained for the nonperturbative (hadronic) bag model results [5]. Furthermore, the frequently used [6, 7, 24, 26] expression for δR^-_s in (8) due to a strangeness asymmetry represents already a priori an overestimate since it results from treating naively the CC transition $\overset{(-)}{s} \to \overset{(-)}{c}$ without a kine-

matic suppression factor for massive charm production [3]. Nevertheless one obtains $\Delta s_W^2|_{\rm strange} = \delta R_s^- = -0.0021$ using [25] $S^- = 0.00165$, instead of $\Delta s_W^2|_{\rm strange} = -0.0017$ in (6), as derived from (4). Therefore the $\delta R_{I,s}^-$ in (8) should be avoided, in particular δR_I^-, and the shift in s_W^2 should rather be evaluated according to (4) corresponding to the actual NuTeV measurements [1].

To summarize, we evaluated the modifications $\delta \overset{(-)}{q}(x, Q^2)$ to the standard isospin symmetric parton distributions due to QED $\mathcal{O}(\alpha)$ photon bremsstrahlung corrections. Predictions are obtained for the isospin violating valence δq_v and sea $\delta \bar{q}$ distributions $(q = u, d)$ within the framework of the dynamical (radiative) parton model. For illustration we compared our radiative QED results for the isospin asymmetries $\delta u_v(x, Q^2)$ and $\delta d_v(x, Q^2)$ with nonperturbative bag model calculations where the violation of isospin symmetry arises from entirely *different* (hadronic) sources, predominantly through quark and target mass differences. Taken together, these two isospin violating effects reduce already significantly the large NuTeV result for $\sin^2\theta_W$. Since, besides isospin asymmetries, the NuTeV group has also disregarded possible effects caused by a strangeness asymmetry $(s \neq \bar{s})$ in their original analysis [1], we have included a recent conservative estimate of the $s \neq \bar{s}$ contribution to $\Delta\sin^2\theta_W$ as well. Together with the isospin violating contributions (cf.(6)), the discrepancy between the large result for $\sin^2\theta_W$ as derived from deep inelastic $\nu(\bar{\nu})N$ data (NuTeV 'anomaly') and the world average of other measurements is entirely removed.

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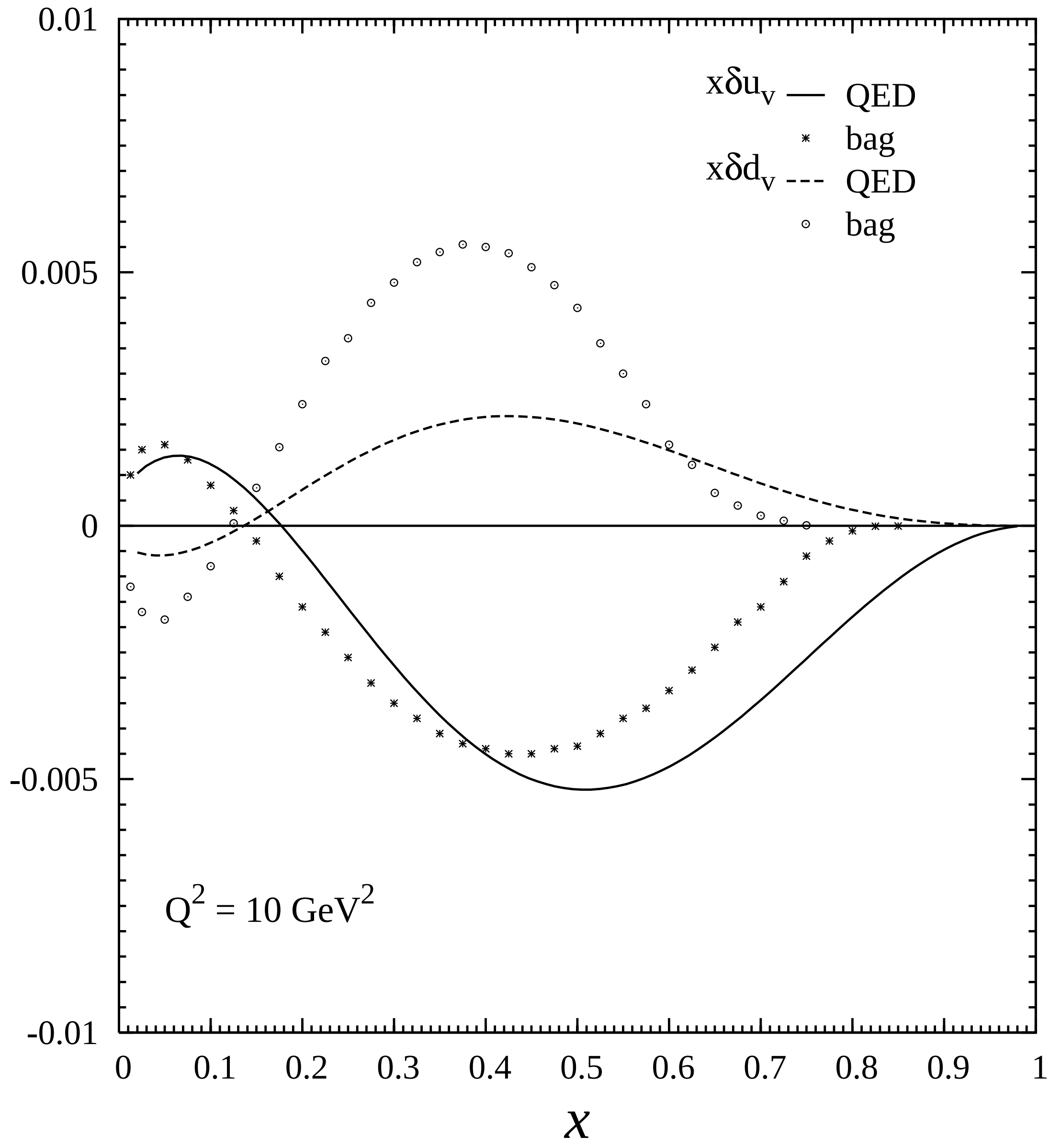

Figure 1: The isospin violating 'majority' δu_v and 'minority' δd_v valence quark distributions at $Q^2 = 10$ GeV^2 as defined in (1). Our radiative QED predictions are calculated according to (3). The bag model estimates are taken from Ref. [5].

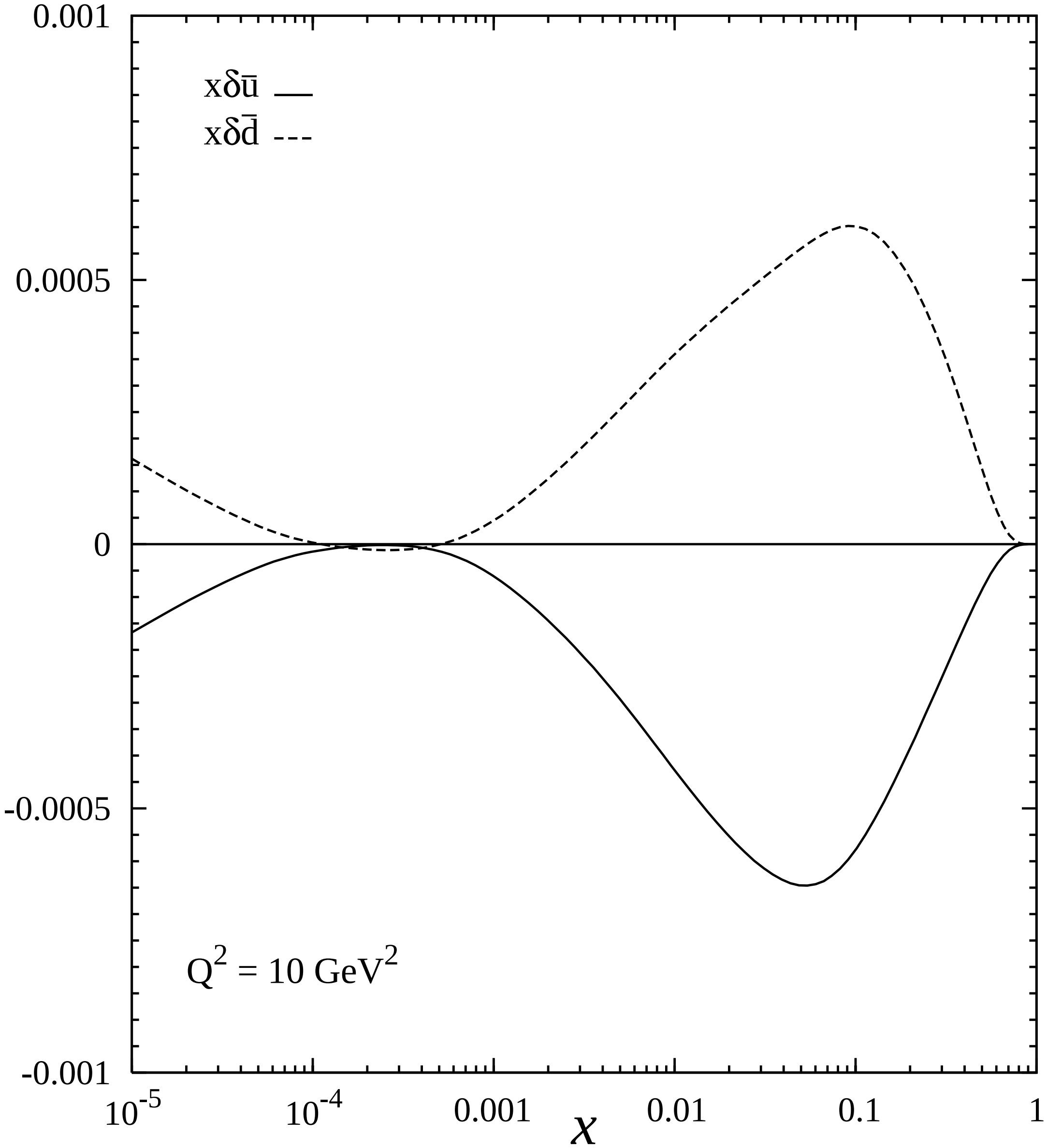

Figure 2: The isospin violating sea distributions $\delta\bar{u}$ and $\delta\bar{d}$ at $Q^2 = 10$ GeV^2 as defined in (1) with u_v, d_v replaced by $\bar{u}$, $\bar{d}$. The QED predictions are calculated according to (3) with u_v, d_v replaced by $\bar{u}$, $\bar{d}$.